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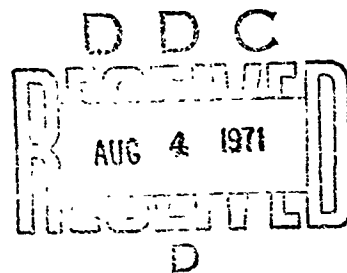
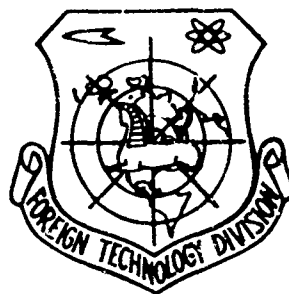
## FOREIGN TECHNOLOGY DIVISION



THEORETICAL PRINCIPLES IN THE DESIGN OF DIGITAL  
INTEGRATING MACHINES WITH  
MULTIDIGIT INCREMENTS

by

A. V. Kalyayev



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## EDITED TRANSLATION

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# THEORETICAL PRINCIPLES IN THE DESIGN OF DIGITAL INTEGRATING MACHINES WITH MULTIDIGIT INCREMENTS

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## 1. A Method for Increasing the Operating Speed and Accuracy of Digital Integrating Machines

The main disadvantage of ordinary digital differential analyzers (DDA) is their low operating speed. We know that we can increase this if we reduce accuracy; however, as a rule, this cannot be done when using a DDA for simulation and control. The low operating speed and relatively low accuracy of the DDA are due to single-digit increments of variables and the rough formula of numerical integration (the formula of rectangles) on which the design of the digital integrators is based.

We can, however, overcome the deficiencies of the DDA and substantially increase its operating speed and, at the same time, accuracy if we use more accurate formulas of numerical integration and multidigit increments in the design of the digital integrators.

Computers consisting of digital integrators and adders, in which multidigit increments and precise formulas of numerical integration are used, will henceforth be called digital integrating machines (DIM) to distinguish them from ordinary digital differential analyzers.

Digital integrating machines enable us to solve a wide range of problems which are important from the theoretical and practical points of view. These machines can be successfully used for real-time digital simulation and for designing accurate high-speed control systems.

Digital integrating machines can be designed in the form of homogeneous computer structures consisting of single-type universal digital integrators which accomplish summation simultaneously with integration. Such computer designs offer high operating speed and sufficiently high accuracy and ensure the necessary reliability as well as continuity of operation during the breakdown of a considerable portion of the differential integrators. They further possess the capacity for rapid program reorganization and make it relative easy to realize digital integrators and switching elements based on microradioelectronic circuits. This last factor enables their mass production and makes it possible to build up the structure by simple connection to the last of the additional digital integrators and commutators, while maintaining small size and weight.

## 2. Algorithms for Digital Integrators, Adders, and Increment Extrapolators

Shannon's system of equations [1], which can be reduced to the following symmetric form, is applied in digital integrating machines:

$$\left. \begin{aligned} dy_{pk} &= \sum_{j=1}^N A_{pkj} dx_j, \\ dy_{qk} &= \sum_{j=1}^N A_{qkj} dx_j, \\ dx_k &= y_{pk} dy_{qk}, \\ dx_1 &= dx, \\ y_{pk}(x_0) &= y_{pk0}, \\ K &= 2, 3, \dots N. \end{aligned} \right\} \quad (1)$$

Here  $A_{pkj}$  and  $A_{qkj}$  ( $K = 1, 2, \dots N$ ;  $j = 1, 2, \dots N$ ) are constant coefficients which take the values 0 or 1 depending upon the problem to be solved. Square matrices, consisting of coefficients  $A_{pkj}$  and  $A_{qkj}$ , along with the vector of initial conditions  $y_{pk0}$  ( $K = 1, 2,$

3 ... N), give completely to Shannon's system of equations (1) and the integrator commutation program in the DIM.

A wide range of problems connected with the processing and operation of digital models and control systems can be reduced to Shannon's system of equations. These problems include ordinary linear and nonlinear differential equations and systems of them with initial and boundary conditions; partial differential equations which can be reduced to ordinary differential equations; transcendental, power, and linear algebraic equations; problems on computing different types of integrals, derivatives, and extrema; problems on computing a function with one or many variables; problems on transforming coordinates; problems involved in the theory of optimal processes, the theory of differential games, and many others.

Shannon's system of equations (1) is characterized by the fact that only the operations of addition and multiplication are used in its right side and none of the more complex operations (division, raising to a power, functional transformation, etc.) is present. This circumstance simplifies the design of the DIM considerably since there is no need to have complex functional converters.

Along with this, in the numerical integration of Shannon's equations (1) there arises the problem of calculating Stieltjes' integral

$$\nabla Z_{x_i+1} = \int_{x_i}^{x_{i+1}} y_{\alpha}(x) dy_{\alpha}(x), \quad (2)$$

which is produced as a result of the integration of equations (1) on the segment from  $x = x_i$  to  $x = x_{i+1}$ .

A detailed study of Stieltjes' process of numerical integration enabled us to obtain corresponding formulas of a different order of accuracy.

In the general case, an interpolation formula of Stieltjes' numerical integration of the n-th order of accuracy can be written in the following form:

$$\begin{aligned} \nabla z_{k(i+1)} &= y_{pk(i+1)} \nabla y_{qk(i+1)} + \frac{1}{2} \nabla y_{pk(i+1)} \nabla y_{qk(i+1)} + \\ &+ \sum_{\alpha=0}^{\frac{2n-9+(-1)^n}{2}} \sum_{\beta=\alpha+1}^{n-\alpha-3} a_{\alpha\beta n} [\nabla y_{pk(i+1-\alpha)} \nabla y_{qk(i+1-\beta)} - \\ &- \nabla y_{pk(i+1-\beta)} \nabla y_{qk(i+1-\alpha)}] \\ &n=4, 5, 6, \dots \end{aligned} \quad (3)$$

Here  $a_{\alpha\beta n}$  are constant coefficients (Table 1).

Table 1. Coefficients  $a_{\alpha\beta n}$ .

$n=4$		$n=5$	
$\beta \backslash \alpha$	0	$\beta \backslash \alpha$	0
1	$-\frac{1}{12}$	1	$-\frac{1}{6}$
2	$+\frac{1}{24}$	2	$+\frac{1}{24}$

$n=6$		$n=7$	
$\beta \backslash \alpha$	0	$\beta \backslash \alpha$	0
1	$-\frac{89}{360}$	1	$-\frac{39}{120}$
2	$+\frac{11}{90}$	2	$+\frac{43}{180}$
3	$-\frac{19}{720}$	3	$-\frac{19}{360}$

The various values of  $n$  given in (3) are obtained by specific Stieltjes numerical integration formulas, which can be used as a basis for synthesis of digital integrators. A distinguishing feature of such integrators is the fact that they produce numerical integration not only with respect to independent variable  $x$  but also with respect to any dependent variable  $y_{qk}(x)$ .

Information is fed to the input of the digital integrators in the form of increments of the integrand  $\nabla y_{pk(i+1)}$  and the integration variable  $\nabla y_{qk(i+1)}$ . These increments can be computed if we integrate in interval  $(x_i, x_{i+1})$  the first two equations of Shannon's system (1):

$$\left. \begin{aligned} \nabla y_{pk(i+1)} &= \sum_{j=1}^N A_{pj} \nabla Z_{kj(i+1)} \\ \nabla y_{qk(i+1)} &= \sum_{j=1}^N A_{qj} \nabla Z_{kj(i+1)} \end{aligned} \right\} \quad (4)$$

The numerical integration formula (3) is an interpolation formula, i.e., it requires, for computation of increments of integral  $\nabla Z_{k(i+1)}$ , the use of increments  $\nabla y_{pk(i+1)}$  and  $\nabla y_{qk(i+1)}$ , which cannot be determined directly since, for this, in accordance with expression (4), it is necessary to have the values of unknown increments  $\nabla Z_{k(i+1)}$ . To eliminate this difficulty, we can use the Stieltjes extrapolation formulas of numerical integration. However, the extrapolation formulas, as detailed study shows, are considerably more complex and less accurate. Therefore, for numerical integration of Shannon's equations, it is better to use interpolation formula (3), while the increments  $\nabla y_{pk(i+1)}$  and  $\nabla y_{qk(i+1)}$  are obtained by extrapolation of increments  $\nabla y_{pki}$  and  $\nabla y_{qki}$  one step forward. More accurately, we should extrapolate increment  $\nabla Z_{ki}$  one step forward, which is done rather simply with the aid of expression

$$\nabla Z_{k(i+1)} = \sum_{a=1}^n (-1)^{a-1} \binom{n}{a} \nabla Z_{k(i+1-a)}. \quad (5)$$

After extrapolation of increments  $\nabla Z_{ki}$  it is easy to calculate, according to formulas (4), increments  $\nabla y_{pk(i+1)}$  and  $\nabla y_{qk(i+1)}$ :

$$\left. \begin{aligned} \nabla y_{pk(i+1)} &= \sum_{j=1}^N A_{pj} \nabla Z_{kj(i+1)}^* \\ \nabla y_{qk(i+1)} &= \sum_{j=1}^N A_{qj} \nabla Z_{kj(i+1)}^* \end{aligned} \right\} \quad (6)$$

Expression (5) is obtained in such a manner that integration error remains invariable.

This shows that, in the general case, a digital integrating machine must consist (Fig. 1) of digital integrators in which Stieltjes' numerical integration formula (3) is applied; increment extrapolators in which increments are extrapolated one step forward in accordance with expressions (5); and adders which form, on the basis of formulas (6), increments of the integrand and the integration variable. These

operations and the solving units corresponding to them can be united into one unified universal digital integrator (Fig. 2).

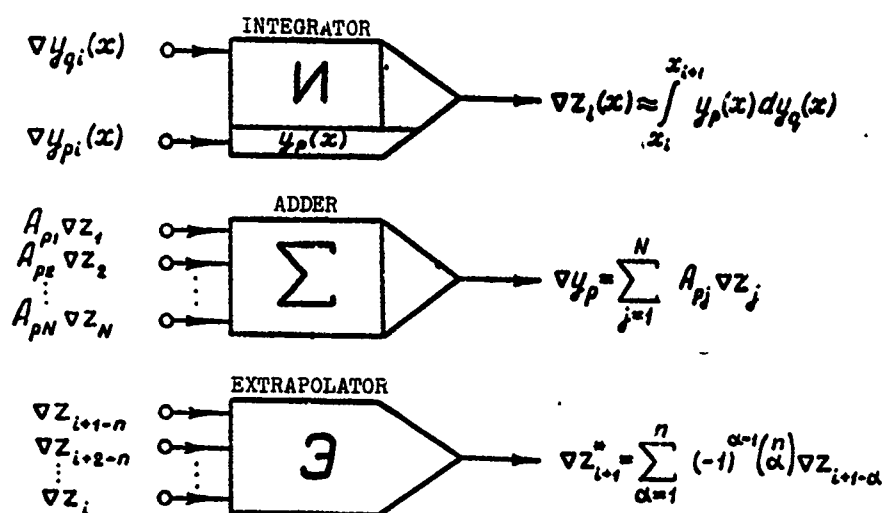


Fig. 1.

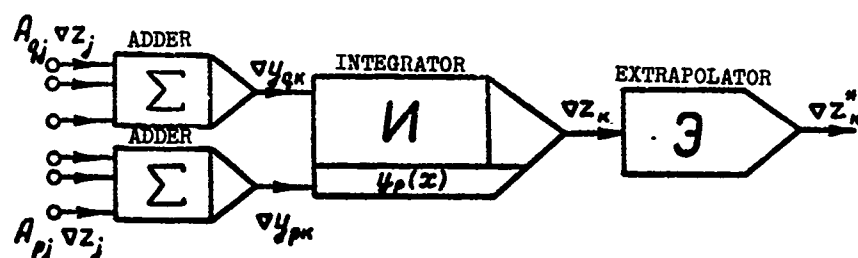


Fig. 2.

On the whole, parallel-type digital integrating machines apply a different network of integration for a symmetric system of Shannon's equations, which is formed by combining expressions (3), (5), and (6).



$$\nabla Z_{k(i+1)}^* = \sum_{a=1}^n (-1)^{a-1} \binom{n}{a} \nabla Z_{k(i+1-a)}^*.$$

$$\nabla y_{pk} = \sum_{j=1}^N A_{pkj} \nabla Z_{k(i+1)}^*.$$

$$\nabla y_{ek} = \sum_{j=1}^N A_{ekj} \nabla Z_{k(i+1)}^*.$$

$$\nabla Z_{k(i+1)} = y_{pk} \nabla y_{ek(i+1)} + \frac{1}{2} \nabla y_{pk(i+1)} \nabla y_{ek(i+1)} +$$

$$+ \sum_{a=0}^{\frac{2n-3+(-1)^n}{4}} \sum_{\beta=a+1}^{n-a-1} a \alpha \beta \alpha [\nabla y_{pk(i+1-a)} \nabla y_{ek(i+1-\beta)} -$$

$$- \nabla y_{pk(i+1-\beta)} \nabla y_{ek(i+1-a)}],$$

$$y_{pk(i+1)} = y_{pk} + \nabla y_{pk(i+1)},$$

$$x_{i+1} = x_i + \nabla x_i,$$

$$y_{pk}(x_0) = y_{pk0},$$

$$K=2, 3, \dots N.$$

(7)

For a serial design DIM, the difference network is somewhat altered due to the fact that, in this case, some of the increments  $\nabla Z_{j(i+1)}$  ( $j = 1, 2, \dots N$ ) are unknown when the integration operation is being performed, and there is no need to find their extrapolated values. This is easily taken into account if we add the following expression to system (7):

$$\begin{aligned} \nabla Z_{k(i+1)}^* &= \nabla Z_{k(i+1)}^* \sigma_1^0(K-j) + \\ &+ \nabla Z_{k(i+1)}^* \sigma_0^1(j-K) \end{aligned} \quad (8)$$

and replace the quantities  $\nabla Z_{j(i+1)}^*$ , which are in (7) under summation signs, with quantities  $\nabla Z_{j(i+1)}^{**}$ . As a result, we obtain a difference network for integrating Shannon's equations for a serial DIM.

The quantities  $\sigma_0^1(\xi)$  and  $\sigma_1^0(\xi)$  in (8) are modernized unit functions

$$\begin{aligned} \sigma_0^1(\xi) &= \begin{cases} 1, & \text{if } \xi \geq 0, \\ 0, & \text{if } \xi < 0, \end{cases} \\ \sigma_1^0(\xi) &= \begin{cases} 1, & \text{if } \xi > 0, \\ 0, & \text{if } \xi \leq 0. \end{cases} \end{aligned} \quad (9)$$

### 3. Error Evaluation for Digital Integrators and Digital Integrating Machines

Since computation is carried out on the basis of approximate formula (3) in the digital integrator, this leads to the appearance of error in the integration method which, for an individual integrator, is evaluated in one integration step by the following expression:

$$\nabla \mu_k(x) = (\nabla x)^n \sum_{j=1}^n b_{j(n-1)} [y_{pk}^{(n-1)}(x) y_{ek}^{(1)}(x) - y_{pk}^{(1)}(x) y_{ek}^{(n-1)}(x)] \quad (10)$$

Constant coefficients  $b_{j(n-1)}$  are presented in Table 2.

Formula (10) shows that the accuracy of the work and, consequently, the operating speed of the digital integrators increases sharply with an increase in the order of accuracy  $n$  for Stieltjes' numerical integration formula (3).

Table 2. Coefficients  $b_{lj}$ .

$j \backslash l$	1	2	3	4	5	6	7
1	$-\frac{1}{2}$	$-\frac{1}{12}$	$-\frac{1}{24}$	$-\frac{19}{720}$	$-\frac{3}{160}$	$-\frac{863}{60480}$	$-\frac{275}{24192}$
2	$\frac{1}{12}$	0	$-\frac{1}{720}$	$-\frac{1}{720}$	$-\frac{5}{4032}$	$-\frac{11}{10080}$	$-\frac{3499}{3628800}$
3	$\frac{1}{24}$	$\frac{1}{720}$	0	$-\frac{1}{3780}$	$-\frac{13}{40320}$	$-\frac{131}{403200}$	$-\frac{161}{518400}$
4	$\frac{19}{720}$	$\frac{1}{720}$	$\frac{1}{3780}$	0	$-\frac{313}{3628800}$	$-\frac{17}{145152}$	$-\frac{30421}{239500800}$
5	$\frac{3}{160}$	$\frac{5}{4032}$	$\frac{13}{40320}$	$\frac{313}{3628800}$	0	$-\frac{551}{14968800}$	$-\frac{25651}{479001600}$
6	$\frac{863}{60480}$	$\frac{11}{10080}$	$\frac{131}{403200}$	$\frac{17}{145152}$	$\frac{551}{14968800}$	0	$-\frac{8067077}{854003200}$
7	$\frac{275}{24192}$	$\frac{3499}{3628800}$	$\frac{161}{518400}$	$\frac{30421}{239500800}$	$\frac{25651}{479001600}$	$\frac{8067077}{854003200}$	0

In addition to method error, quantization error occurs in digital integrators as a result of the representation of variables and increments in the DIM by a finite number of positions. For an individual integrator, in one integration step the quantization error is

$$\nabla \beta_k(x) = y_{pk}(x)[O_{pk}(x) - O_{pk}(x - \nabla x)] + \nabla y_{qk}(x)O_{pk}(x - \nabla x). \quad (11)$$

Here  $O_{qk}(x)$  and  $O_{pk}(x)$  are remainders which are dropped when rounding off variables  $y_{qk}(x)$  and  $y_{pk}(x)$ , while increment  $\nabla y_{qk}(x)$  is the difference  $\nabla y_{qk}(x) = y_{qk}(x) - y_{qk}(x - \nabla x)$ .

The total error in one integration step for an individual digital integrator consists of the sum of the method and quantization errors:

$$\nabla \epsilon_k(x) = \nabla \mu_k(x) + \nabla \beta_k(x). \quad (12)$$

The order of accuracy for the numerical integration formula  $n$ , on which method error depends, and the number of quanta  $\nabla x$ ,  $\nabla y_{pk}$ ,  $\nabla y_{qk}$ , and  $\nabla Z_k$ , on which quantization error depends, must be selected in such a manner that errors  $\nabla \mu_k(x)$  and  $\nabla \beta_k(x)$  have the same order of smallness. Only when this condition is fulfilled are DIM equipment costs minimal and operating speed and accuracy maximal.

In order to evaluate the operational accuracy of a digital integrating machine on the whole, the error formed in the full integration range  $(x - x_0)$  is more significant. Total error for the DIM is determined by the following system of differential equations:

$$\left. \begin{aligned} de_{\mu k}(x) &= \sum_{j=1}^N A_{kj} de_{\mu j}(x), \\ de_{\beta k}(x) &= \sum_{j=1}^N A_{kj} de_{\beta j}(x), \\ de_{\mu k}(x) &= y_{pk}(x) de_{qk}(x) + \\ &+ e_{pk}(x) dy_{qk}(x) + de_{\beta k}(x), \\ e_{\mu k}(x_0) &= 0, \\ K &= 2, 3, \dots N. \end{aligned} \right\} \quad (13)$$

Here

$$\begin{aligned} de_{\mu k}(x) &= (\nabla x)^{n-1} \sum_{i=1}^n b_{ni-k} [y_{pk}^{(n-i)}(x) y_{qk}^{(i)}(x) - \\ &- y_{qk}^{(i)}(x) y_{pk}^{(n-i)}(x)] dx + [y_{pk}(x) O'_{qk}(x) + \\ &+ y'_{qk}(x) O_{pk}(x)] dx \end{aligned} \quad (14)$$

is the internal differential error of the digital integrator, and the quantities  $\epsilon_{pk}(x)$ ,  $\epsilon_{qk}(x)$  and  $\epsilon_{zk}(x)$  are errors of functions  $y_{pk}(x)$ ,  $y_{qk}(x)$  and  $z_k(x)$  in the large integration range  $(x - x_0)$ . Functions  $y_{pk}(x)$  and  $y_{qk}(x)$ , when equations (13) are solved, are considered given. If functions  $y_{pk}(x)$  and  $y_{qk}(x)$  are known beforehand, as, for example, when reproducing functional relationships in a DIM, the solution to equations (13) can be obtained, in some cases, in explicit form, i.e., in the form of formulas for errors of specific variables.

#### 4. The Dependence of DIM Operating Speed and Accuracy on Stieltjes' Numerical Integration Formula

A variation in the order of accuracy of Stieltjes' numerical integration formula (3) affects only method error and has virtually no effect on quantization error. Therefore, when evaluating operating speed and accuracy of a DIM as a function of the order of accuracy of the numerical integration formulas (3), we should consider only method error (10). In this case, as follows from (10), the internal differential error of the digital integrator (14) will depend upon the integration step  $\nabla x$ , the order of accuracy of the integration formula  $n$ , and the independent machine variable  $x$

$$de_k(x, n, \nabla x) = (\nabla x)^{n-1} \varphi_k(x, n) dx. \quad (15)$$

Due to the fact that equations (13), determining DIM error, are linear, the dependence of errors  $\epsilon_{pk}$ ,  $\epsilon_{qk}$ , and  $\epsilon_{zk}$  on quantities  $\nabla x$ ,  $x$  and  $n$  will be similar to relationship (15):

$$\epsilon_{pk} = (\nabla x)^{n-1} \Psi_{pk}(x, n). \quad (16)$$

In order to evaluate the accuracy and operating speed of the DIM, it is more convenient to use not the instantaneous values, but the mean-square values of errors

$$\epsilon_{pk}^2 = (\nabla x)^{2n-2} \Psi_{pk}^2(x, n), \quad (17)$$

where

$$\Psi_{pk}^2(x, n) = \frac{1}{x} \int_{x_0}^x \Psi_{pk}^2(x, n) dx. \quad (18)$$

Let us examine the transition from Stieltjes' numerical integration formula of the  $m$ -th order of accuracy to a formula of the  $n$ -th order of accuracy, and let us evaluate the change in operating speed and error. We shall assume that at the same time the numerical integration formula changes, the integration step changes from  $\nabla_m x$  to  $\nabla_n x$ . The ratio of the mean-square error

$$\eta = \frac{(\nabla_m x)^{m-1} \Psi_{pk}^*(x, m)}{(\nabla_n x)^{n-1} \Psi_{pk}^*(x, n)} \quad (19)$$

determines the increase in DIM accuracy with the change from a numerical integration formula of order  $m$  to a formula of order  $n$ .

In turn, ratio

$$C = \frac{\nabla_n x}{\nabla_m x} \quad (20)$$

evaluates the change in operating speed when the integration step changes from  $\nabla_m x$  to  $\nabla_n x$ . From (19) and (20) we obtain

$$\eta = \frac{(\nabla_m x)^{m-1} \Psi_{pk}(x, m, n)}{C^{n-1}} \quad (21)$$

where

$$\Psi_{pk}(x, m, n) = \frac{\Psi_{pk}^*(x, m)}{\Psi_{pk}^*(x, n)} \quad (22)$$

Formula (21) determines the connection between operating speed and operational accuracy of the DIM during transition from one numerical integration formula to another and during the simultaneous change in integration step.

Function  $\Psi_{pk}(x, m, n)$  can be evaluated if we give a frequency pass band for the DIM of  $\omega_c$ , i.e., maximum harmonic frequency of the functions applied in the machine. Let us examine the most adverse case, when all integrands have the frequency of the highest harmonic

$$y_{pk}(x) = a_k \sin \omega_c x,$$

and the integration variables agree with the independent machine variable  $y_{qk}(x) = x$ . It is easy to see that, in this case,

$$\Psi_{pk}(x, m, n) = \frac{b_{1(m-1)}}{b_{1(n-1)}} \omega_c^{m-n} \quad (23)$$

where the coefficients  $b_{1(m-1)}$  and  $b_{1(n-1)}$  are determined by Table 2.

Taking into account (23), the change in operating speed and accuracy during transition from one numerical integration formula to another can be evaluated with the aid of expression

$$\eta = \frac{b_{1(m-1)}}{b_{1(n-1)}} \cdot \frac{(\omega_s \nabla_m x)^{m-2}}{C^{n-1}}. \quad (24)$$

Let us examine certain particular cases of change in DIM operating speed and accuracy.

From expression (24) we can obtain a formula connecting the accuracy of the DIM with the integration step under the condition that operating speed is maintained ( $C = 1$ )

$$\eta = \frac{b_{1(m-1)}}{b_{1(n-1)}} (\omega_s \nabla x)^{m-2}. \quad (25)$$

Here  $m$  for  $\nabla x$  is omitted since the integration step, in this case, is kept invariable.

A formula connecting operating speed and integration step, with accuracy preserved ( $\eta = 1$ ), can also be obtained based on (24):

$$C = \left[ \frac{b_{1(m-1)}}{b_{1(n-1)}} \right]^{\frac{1}{n-1}} (\omega_s \nabla_m x)^{\frac{m-2}{n-1}} \quad (26)$$

Finally, we can derive an expression connecting operating speed and accuracy when the integration step is changed but the numerical integration formula ( $m = n$ ) is preserved. Taking into account that when  $m = n$ ,  $b_{1(m-1)} = b_{1(n-1)}$ , we have

$$C = \frac{1}{(\eta)^{\frac{1}{n-1}}}. \quad (27)$$

For an ordinary DDA operating on the formula of rectangles ( $n = 2$ ), we have the known relationship

$$C = \frac{1}{\eta}. \quad (28)$$

Analysis of expressions connecting the operating speed and accuracy of a DIM indicate that owing to the change to Stieltjes' numerical integration formulas of a higher order, we can substantially improve the accuracy of the DIM while preserving operating speed, or considerably increase operating speed while preserving accuracy. At the same time, we can also improve (by several orders of magnitude as compared with a DDA) both the operating speed and accuracy of a DIM.

The disadvantage of a DDA is the fact that when problem-solving accuracy is increased, solution rate must decrease in inverse proportion, in accordance with expression (28). In a DIM, in which more accurate numerical integration formulas are used, the relationship between operating speed and variation in solution accuracy, as expression (27) shows, is weaker, which is a definite advantage of a DIM.

#### 5. Optimal Stieltjes' Numerical Integration Formulas

From the set of numerical integration formulas (3) we should select optimal formulas from the point of view of accuracy, operating speed, and equipment cost. For this purpose, let us compare Stieltjes' numerical integration formulas having a high order of accuracy ( $n \geq 3$ ) with the formula of rectangles ( $m = 2$ ). Based on expression (24), for  $m = 2$  we can write

$$C \cdot \eta^{\frac{1}{n-1}} = \frac{(\omega_s \nabla_2 x)^{\frac{2-n}{n-1}}}{[2b_{1(n-1)}]^{\frac{1}{n-1}}} \approx 2(\omega_s \nabla_2 x)^{\frac{2-n}{n-1}}. \quad (29)$$

When changing from a less accurate numerical integration formula to a more accurate formula, not only accuracy and speed increase, but also the amount of DIM equipment increases. The relative increase in DIM equipment during a transition from the formula of rectangles to a formula of the  $n$ -th order of accuracy is evaluated by expression

$$Q = \frac{3}{4} \left[ (n-2)^2 - \frac{1-(-1)^n}{2} \right] + 2, \quad n \geq 3. \quad (30)$$

Let us examine the ratio of quantities  $C \cdot \eta^{\frac{1}{n-1}}$  and  $Q$ , which characterizes, to a certain extent, the increase in accuracy and speed of a DIM per unit of equipment when the accuracy of the integration formula increases:

$$\theta(n) = \frac{C \cdot \eta^{\frac{1}{n-1}}}{Q} = \frac{8(\omega_0 \nabla x)^{\frac{2-n}{n-1}}}{3 \left[ (n-2)^2 - \frac{1-(-1)^n}{2} \right] + 8}. \quad (31)$$

Quantity  $\theta(n)$  can be interpreted as the coefficient of quality or the coefficient of information output for the DIM. In order to achieve the fastest operating speed and the maximum accuracy per unit of DIM equipment, it is necessary to strive for the maximum value of the quality coefficient. Analysis shows that function  $\theta(n)$  has a maximum at  $n \approx 4-5$ . Consequently, the most optimal formula, from the point of view of highest operating speed, maximum accuracy, and minimum equipment cost, is Stieltjes' numerical integration formula when  $n = 4$ , i.e., the formula of quadratic parabolas:

$$\begin{aligned} \nabla Z_{n+1} &= y_{n+1} \nabla y_{n+1} + \frac{1}{2} \nabla y_{n+1} \nabla y_{n+1} + \\ &+ \frac{1}{12} [\nabla y_{n+1} \nabla y_{n+1} - \nabla y_{n+1} \nabla y_{n+1}]. \end{aligned} \quad (32)$$

The corresponding extrapolation formula then acquires the form

$$\begin{aligned} \nabla Z_{n+1} &= 4 \nabla Z_n - 6 \nabla Z_{n-1} + \\ &+ 4 \nabla Z_{n-2} - \nabla Z_{n-3} \end{aligned} \quad (33)$$

Method error in one integration step for formula (32) can be evaluated by the following expression

$$\begin{aligned} \nabla \mu_n(x) &= (\nabla x)^4 \cdot \frac{1}{24} [y_{n+1}^{(4)}(x) y_{n+1}'(x) - \\ &- y_{n+1}'(x) y_{n+1}^{(4)}(x)]. \end{aligned} \quad (34)$$

Quantization error of the formula of quadratic parabolas is determined by general expression (11), which is identical for formulas of any order of accuracy.



## 6. Discussion of Results

This study shows that on the basis of Stieltjes' numerical integration formulas (3), digital integrating machines can be designed which significantly surpass ordinary digital differential analyzers in accuracy and operating speed. A DIM based on formulas of quadratic (32) or cubic parabolas offers the greatest information output per unit of equipment cost. In this respect, digital differential analyzers based on the formula of rectangles are the poorest version of DIM and can be used only when it is necessary to reduce the equipment cost to minimum, and operating speed and accuracy do not have deciding values. When maximum accuracy and the highest operating speed must be achieved at the same time, while retaining the smallest dimensions and weight, Stieltjes' numerical integration formulas (3), which have been presented in this work, should be the basis for designing digital integrating machines. These formulas make it possible to design compact DIM, suitable for work in real-time computation, which ensure sufficiently high accuracy and have relatively small size and weight.

## References

1. Shannon, C. Mathematical Theory of the Differential Analyzer. J. Math. and Phys. 20, No. 4, 1941.

## Summary

A modernized version of Shannon's systems of differential equations underlies the design of digital integrating machines (DIM) with multidigit increments. The integration of Shannon's equations is accomplished by means of Stieltjes' numerical integration formulas.

Digital integrators which operate with multidigit increments have been constructed on the basis of Stieltjes' numerical integration formulas. It is demonstrated that any system of Shannon's equations can be solved with the use of computing units of three types: digital integrators, adders, and extrapolators of increments.

Errors of digital integrators are analyzed. Errors of method and quantization are examined. A general evaluation of errors of digital integrating machines is made.

Operating speed, accuracy, and complexity of digital integrating machines are studied depending upon the numerical integration formulas which underlie the construction of the digital integrators. The optimal connection between numerical integration and increment extrapolation is studied, and the possible applications of digital integrating machines are examined particularly for real-time computation.

#### Discussion

- H. Seiedrazy: What is the communicational system in multidigit increment computers? Is it ternary or is it a fraction of a word?
- A. V. Kalyayev: Data is transmitted between solving units of digital integrating machines in the form of multidigit increments which consist of several bits.
- H. Seiedrazy: If the output of an integrator  $Z_{K(i+1)}$  is a fraction of a word, such as 8 or 10 bits of a total of 30 bits, what are the factors involved in choosing this fraction of a word for  $Z_{K(i+1)}$  and what is the effect of this selection on quantization and method error?
- A. V. Kalyayev: The number of bits of multidigit increments is selected on the condition that method and quantization errors are equal. Digital integrating machines ensure the optimal relationship between accuracy, speed, and equipment cost only when these errors are equal.

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